

Directed Learning Activity

Normal Distribution

1. Inequalities
2. Standard normal distribution table or z table

Objective 1: Inequalities

Many problems in statistics involve translating the problems into inequalities. In this activity, we will learn how to translate phrases into inequalities. We will also learn how to graph inequalities on a number line. When solving problems that are modeled by inequalities, we need to look for key words.

word or phrase	inequality symbol	word or phrase	inequality symbol
at least	\geq	at most	\leq
no less than	\geq	no more than	\leq
more than	$>$	fewer than	$<$
greater than	$>$	less than	$<$

Example:

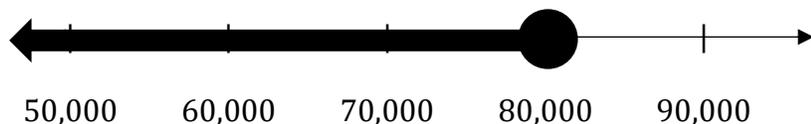
Translate the given statement into an inequality using X as the unknown number then graph it into a number line.

1. John's salary this year will be at most \$80,000.

Solution:

$$X \leq 80,000 \quad (\text{all numbers less than or equal to } 80,000)$$

Let us graph the inequality on a number line. Since 80,000 is included, we will use a shaded circle. Shade the region to the left of 80,000

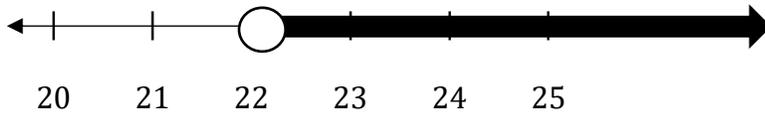


2. Maria is more than 22 years old.

Solution:

$$X > 22 \quad (\text{all numbers greater than 22})$$

Since 22 is not included, use an open circle. Shade the region to the right of 22.



Your turn:

Translate the given statement into an inequality using X as the unknown number then graph it into a number line.

1. Elizabeth must earn at least 520 points to pass the class

2. There were no more than 25 kids in the classroom

3. The cost of the television is at least \$900

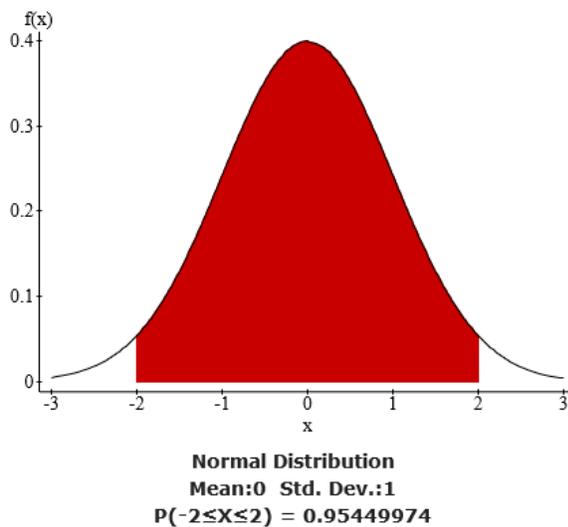
4. There are less than 50 cookies in the cookie jar.

Objective 2: Standard normal distribution table or z table

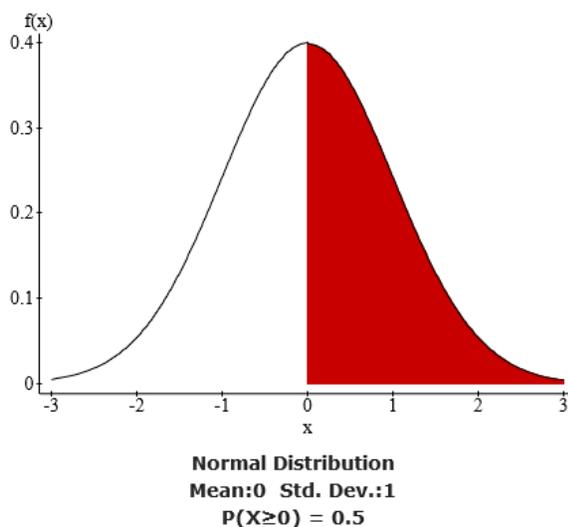
In Statistics, a table that is often used to compute probability, construct confidence interval, hypothesis testing, etc. is the standard normal distribution table of z table. In this next activity, we will learn how to read the table.

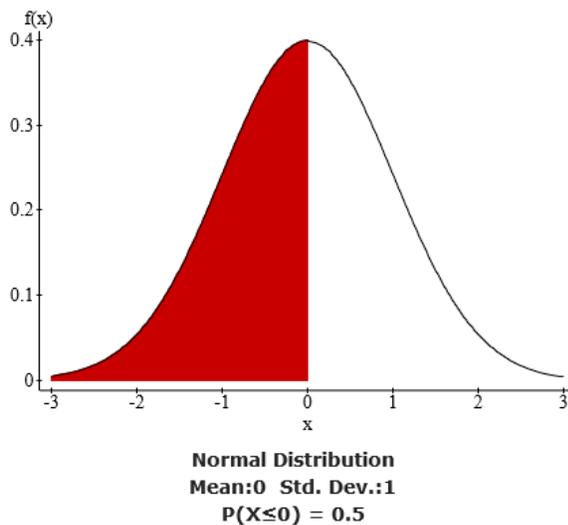
Let us start by discussing some basic information of the standard normal distribution curve.

1. The standard normal distribution curve is bell-shaped. The figure below shows a standard normal curve whose area between -2 and 2 , denoted $-2 < z < 2$, is 0.9545 .



2. The standard normal curve has a mean $\mu = 0$ (located in the center of the curve) and standard deviation $\sigma = 1$. The area to the right of 0, denoted $z > 0$, is 0.5. The area to the left of 0, denoted $z < 0$, is also 0.5. Thus, the **total area under the curve is 1**.





Finding the Area

Let us now learn how to use the standard normal distribution table (See Table E) to find the area under the standard normal curve.

1. The numbers in the left-most column are called the **z-scores**. The left-most column gives us the z-score up to the first decimal places. The decimal digits on the top row are the second decimal places of the z-scores.

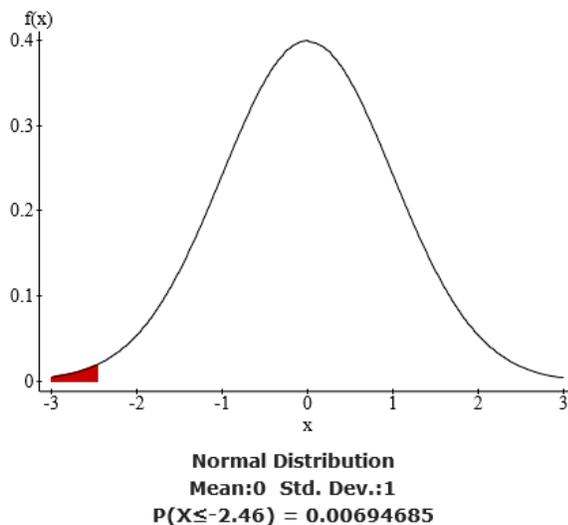
For example, to locate $z = -2.53$.

Find -2.5 on the left most column then $.03$ on the top row.

2. The 4-digit numbers in the middle of the table are the area to the **LEFT** of a z-score.

For example, let us find the area to the left of -2.46 , denoted $z < -2.46$.

Locate -2.4 on the left-most column then $.06$ on the top row. When we cross-reference, they intersect at 0.0069. Thus, the area to the left of -2.46 is **0.0069**.



Note: In normal distribution, the inequalities $z > 1.52$ and $z \geq 1.52$ are equivalent. In other words, the area is the same for both whether the endpoint is included or not.

Examples:

1. Area to the left of $z = 1.26$.

a. Write the inequality.

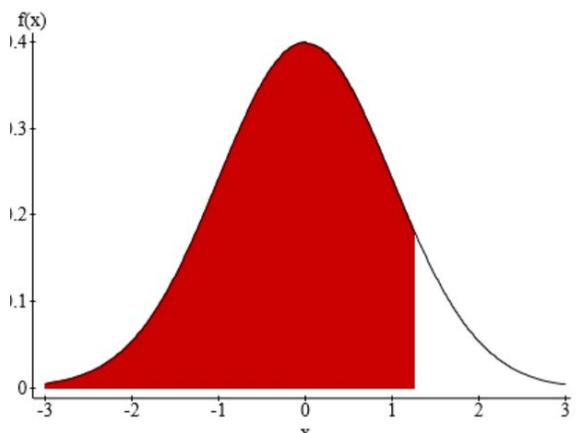
b. Draw the bell-shaped curve and shade the appropriate area.

c. Find the area.

Solution:

a. $z < 1.26$ or $z \leq 1.26$ “area to the left, use $<$ or \leq ”

b. With 0 in the middle, shade the region to the left of 1.26.



c. For positive z-scores, use the second page of the table. Locate 1.2 on the left-most column then .06 on the top row. Area is **0.8962**.

2. Area to the right of $z = -2.34$.

a. Write the inequality.

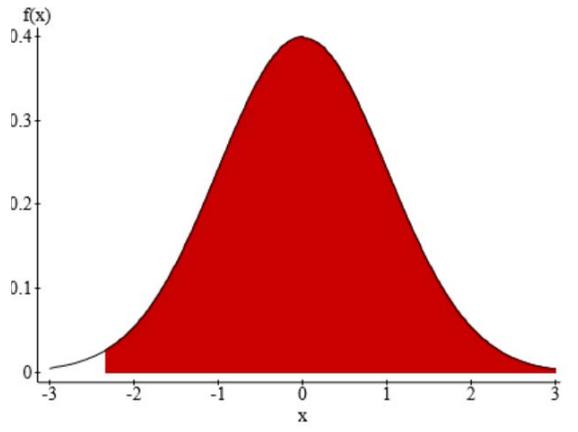
b. Draw the bell-shaped curve and shade the appropriate area.

c. Find the area.

Solution:

a. $z > -2.34$ or $z \geq -2.34$ “area to the right, use $>$ or \geq ”

b. With 0 in the middle, shade the region to the right of -2.34.



c. *Note: The table we are using provides the area to the LEFT of a given z score.*

To find the area to the RIGHT, we will need to subtract it from 1. The area to the left of -2.34 is 0.0096. Thus, the area to the right is $1 - 0.0096 = \mathbf{0.9904}$.

Your turn:

1. Area to the left of $z = -1.08$.

a. Write the inequality.

b. Draw the bell-shaped curve and shade the appropriate area.

c. Find the area.

2. Area to the right of $z = -2.17$.

a. Write the inequality.

b. Draw the bell-shaped curve and shade the appropriate area.

c. Find the area.

3. Area between $z = -2.37$ and $z = 0.97$.

a. Write the inequality.

b. Draw the bell-shaped curve and shade the appropriate area.

c. Find the area.

4. Area between $z = 0.52$ and $z = 2.56$.

a. Write the inequality.

b. Draw the bell-shaped curve and shade the appropriate area.

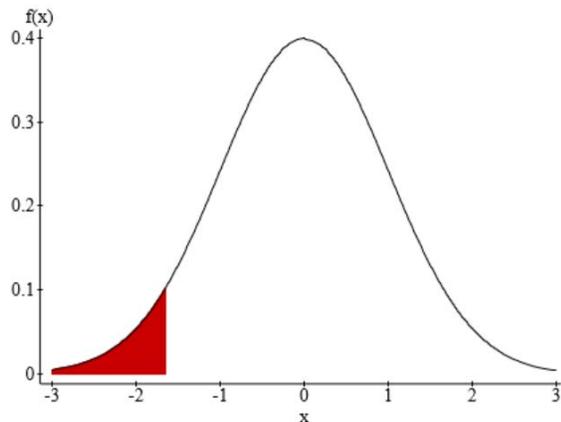
c. Find the area.

Finding the z-score

In this final activity, we will reverse the process. This time we will find the z-score given the area.

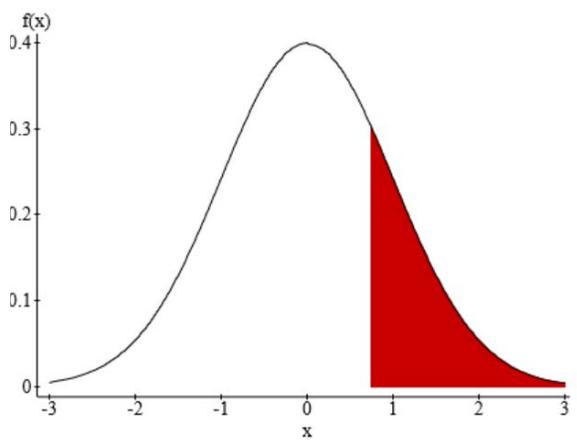
Example:

1. Find the z score whose area to the left is 0.0505.



Solution: Since the area to the left is less than 0.5, we know that the z-score must be negative. In the first page of the table, search for 0.0505 (use the closest number if it is not listed). The z-score that corresponds to the given area is $z = -1.64$.

2. Find the z score whose area to the right is 0.2296.



Solution: We know that the z-score is positive. If the area to the right is 0.2296, it means that the area to the left is $1 - 0.2296 = 0.7704$. Search for 0.7704 in the second page of the Table. The z-score that corresponds to the given area is $z = 0.74$.

Your turn:

5. Find the z-score whose area to the left is 0.8051. (Hint: Draw a picture first)

6. Find the z-score whose area to the right is 0.0287.

7. Find the two z-scores whose area in the middle is 0.75. (Hint: See the figure below)

